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Massless Monopoles Via Confining Phase Superpotentials

S. Elitzur [a 1], A. Forge [a 2], A. Giveon [b 3], K. Intriligator [c 4], E. Rabinovici [a 5]

a Racah Institute of Physics, The Hebrew University
Jerusalem, 91904, Israel

b Theory Division, CERN, CH 1211, Geneva 23, Switzerland

c Institute for Advanced Study, Princeton, NJ 08540, USA

We discuss how the structure of massless monopoles in supersymmetric theories with a Coulomb phase can be obtained from effective superpotentials for a phase with a confined photon. To illustrate the technique, we derive effective superpotentials which can be used to derive the curves which describe the Coulomb phase of $N = 2$, $SU(N_c)$ gauge theory with $N_f < N_c$ flavors.

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[1] e-mail address: elitzur@vms.huji.ac.il

[2] e-mail address: forge@vms.huji.ac.il

[3] On leave of absence from Racah Institute of Physics, The Hebrew University, Jerusalem 91904, Israel; e-mail address: giveon@vxcern.cern.ch

[4] e-mail address keni@sns.ias.edu

[5] e-mail address: eliezer@vms.huji.ac.il

Exact results reveal that supersymmetric theories with a Coulomb phase can have massless monopoles and/or dyons at strong coupling. This was first analyzed in the context of $N = 2$ supersymmetric theories with gauge group $SU(2)$, where it was found that the photon coupling is given by the modulus of a curve which degenerates at the massless monopole points [1]. A similar situation occurs for $N = 1$ supersymmetric theories with a Coulomb phase [2,3,4]. Curves which give the structure of massless monopoles and dyons have been conjectured for a number of different $N = 2$ theories [5-19] and have been shown to pass a number of highly non-trivial checks. In this note we discuss certain effective superpotentials which could be used to *derive* the locus of vacua with massless monopoles dyons and the curves which give the Coulomb gauge couplings.

Our starting point is a supersymmetric theory with a Coulomb phase, which we drive to a partially confining phase by perturbing by a tree level superpotential. For example, we could consider an $N = 2$ supersymmetric theory, which we break to $N = 1$ by adding the tree level superpotential $W_{N=1} = \sum_k g_k U_k$, where U_k are the Casimirs built out of the adjoint Φ of the vector multiplet. At the classical level, there are a number of vacua with different expectation values of Φ, breaking the gauge group to a subgroup in which the adjoint Φ is generically massive. The dynamics of a vacuum with a classically unbroken subgroup $H \times U(1)^l$, with H non-Abelian, is governed by the low energy $N = 1$ theory with gauge group H, which is confined or Higgsed, with a vacuum degeneracy given by gaugino condensation or by a low energy superpotential. The l photons remain massless and decoupled. These phenomena must be reproduced upon perturbing the low energy $N = 2$ theory by $W_{N=1}$.

The spectrum of the low energy $N = 2$ theory for a generic vacuum on the Coulomb branch consists of the $R = rank(G)$ fields U_k and their associated photons. Perturbing such a vacuum by $W_{N=1}$ does not lead to a ground state. The supersymmetric ground states obtained upon perturbing by $W_{N=1}$ only occur where monopoles or dyons become massless and can condense. Consider, for example, the vicinity near where a single monopole or dyon becomes massless. The low energy theory has a superpotential which is approximately given by

$$W = M(U_k) q\tilde{q} + \sum_k g_k U_k \tag{1}$$

and there is a supersymmetric ground state if there is a solution $\langle U_k \rangle$ with

$$M(\langle U_k \rangle) = 0 \qquad \text{and} \qquad \partial_k M(\langle U_k \rangle) \langle q\tilde{q} \rangle = -g_k. \tag{2}$$

In such a vacuum, the condensate $\langle q\widetilde{q}\rangle$ confines one of the photons and $R-1$ are left massless. These vacua must correspond to the vacua in which the gauge group is classically broken to $SU(2)\times U(1)^{R-1}$. There are other vacua in which more than one mutually local monopole condenses, confining more than one photon. These vacua correspond to the classical vacua with an enhanced gauge group which is larger than $SU(2)$. In this letter we will focus on the vacua with a single confined photon, corresponding to the classical $SU(2)\times U(1)^{R-1}$ vacua.

By finding the classical vacua in which the gauge group is broken to $SU(2)\times U(1)^{R-1}$ and analyzing the quantum effects associated with the low energy, $N=1$ supersymmetric $SU(2)$ theory, we can determine where (2) must have a solution – i.e which vacua have a massless monopole or dyon. This is sufficient information to derive the elliptic curve for the gauge couplings. This "integrating in" technique [20,2] has been used to derive the elliptic curves for theories with gauge group $SU(2)$ [2,3,4]. Aspects of the $SU(N_c)$ case were discussed in [4], which considered on the vacua in which all of the photons are confined, corresponding to the classical vacua in which $SU(N_c)$ is unbroken.

Consider, for example, $N=2$, $SU(3)$ Yang-Mills theory. Perturbing by $W_{N=1} = mu+gv$, where $u=\frac{1}{2}\mathrm{tr}\Phi^2$ and $v=\frac{1}{3}\mathrm{tr}\Phi^3$, leads to classical vacua with $\Phi=0$, in which $SU(3)$ is unbroken, and $\Phi=\mathrm{diag}(m/g,m/g,-2m/g)$, in which there is a classically unbroken $SU(2)\times U(1)$. We study the vacuum with unbroken $SU(2)\times U(1)$. The low energy theory in this vacuum consists of a decoupled $N=1$ photon multiplet along with $N=1$, $SU(2)$ Yang-Mills theory with a scale Λ_2 which is related to the scale Λ of the high-energy $N=2$, $SU(3)$ theory by

$$\Lambda_2^6=(3m/g)^{-2}\cdot(3m)^2\Lambda^6=g^2\Lambda^6, \tag{3}$$

where the first factor comes from matching $SU(3)$ to $SU(2)$ at the scale $(m/g)-(-2m/g)=(3m/g)$ of the massive $SU(3)/SU(2)$ W bosons and the second factor comes from matching at the mass $W''(m/g)=(3m)$ of the massive adjoint. The superpotential of the low energy theory in this vacuum is

$$W_L=\frac{m^3}{g^2}\pm 2g\Lambda^3, \tag{4}$$

where the first term is the tree level term $W_{N=1}$ evaluated for $\Phi=\mathrm{diag}(m/g,m/g,-2m/g)$ and the second term is the contribution from gaugino condensation in the unbroken $N=1$ $SU(2)$, with the sign reflecting the vacuum degeneracy.

The superpotential (4) is certainly correct in the limit $m \gg \Lambda$ and $m/g \gg \Lambda$, where the original theory is broken to our low energy theory at a very high scale. We will assume that (4) is exact for all values of the parameters. This assumption is referred to as the assumption of vanishing W_Δ [20]. In some cases, it is possible to prove this assumption [20,2,3,4]. In the case of (4), however, we can not directly rule out additive corrections of the form $W_\Delta = \sum_{n=1}^{\infty} a_n (m^3/g^2)(g\Lambda/m)^{6n}$. (In particular, the condition that the dynamical superpotential of the high-energy theory vanishes is ensured by a $U(1)_R$ symmetry, placing no further constraint on W_Δ.) Assuming that such terms are absent, we will see that we derive results which agree with those of [5,6]; conversely, it is possible to show that the hyper-elliptic curves of [5,6] are obtained only if $W_\Delta = 0$. Physically, the statement that $W_\Delta = 0$ reflects the absence of various operator mixings in the flow to the infrared – perhaps it is related to some sort of integrability. In any case, it is a necessary condition for obtaining our results by "integrating in."

The superpotential (4) implies that there will be two vacua, each with an unconfined photon, at

$$\langle u \rangle = \partial_m W_L = 3\left(\frac{m}{g}\right)^2 \qquad \langle v \rangle = \partial_g W_L = -2\left(\frac{m}{g}\right)^3 \pm 2\Lambda^3. \tag{5}$$

The vacua (5) must be solutions of (2); in particular, these vacua must parameterize the space of massless dyons for $N=2$, $SU(3)$ Yang-Mills theory. The vacua (5) parameterize the singularities of the curve $y^2 = (x^3 - xu - v)^2 - 4\Lambda^6$; we have thus re-derived the result of [5,6] (our normalization for Λ is that appropriate for the $\overline{DR}$ scheme). In exactly the present context, a qualitative argument was given in [9] that this curve must provide two solutions to the equations (2). It is possible to use the explicit expression in [6] for the one-form λ to verify that (5) do indeed solve all equations in (2). Again, we have derived the curve from (5) rather than visa-versa.

This analysis can be directly extended, for example, to $N=2$ supersymmetric $SU(N_c)$ Yang-Mills theory. Upon adding $W_{N=1} = \sum_{r=2}^{N_c} g_r u_r$, with $u_r = \mathrm{tr}\Phi^r/r$, the eigenvalues of Φ are the roots of $W'_{N=1}(x) = g_{N_c}\prod_{i=1}^{N_c-1}(x-a_i)$. The vacua with classical $SU(2)\times U(1)^{N_c-2}$ are those with, say, two eigenvalues equal to a_1 and the rest given by $a_2, \ldots a_{N_c-1}$. The scale Λ_2 of the low energy $N=1$, $SU(2)$ theory is related to the scale Λ of the original $N=2$, $SU(N_c)$ theory by

$$\Lambda_2^6 = \prod_{i\neq 1}(a_1 - a_i)^{-2} \cdot g_{N_c}^2 \prod_{i\neq 1}(a_1 - a_i)^2 \Lambda^{2N_c} = g_{N_c}^2 \Lambda^{2N_c}, \tag{6}$$

where the first factor again reflects the matching at the scale of the $SU(N_c)/SU(2)$ W bosons and the second factor reflects the mass $W''(a_1)$ of the adjoint in this vacuum. Such matching relations appeared in [21]. The superpotential in the low energy theory is

$$W_L = W_{cl}(g) \pm 2 g_{N_c} \Lambda^{N_c}, \tag{7}$$

where $W_{cl}(g)$ is $W_{N=1}$ evaluated in the classical $SU(2) \times U(1)^{N_c-2}$ vacua, and with the second term generated by gaugino condensation in the low energy $SU(2)$ theory. The superpotential (7) implies that the vacua with one photon confined and $N_c - 2$ left unconfined are at

$$\langle u_r \rangle = u_r^{cl}(g) \pm 2\Lambda^{N_c} \delta_{r,N_c}. \tag{8}$$

Defining $P(x; u_r) = \det(x - \Phi)$, there is a solution x^* to $P = P' = 0$ for $u_r = u_r^{cl}(g)$. Because $P(x; \langle u_r \rangle) = P(x; u_r^{cl}(g)) \pm 2\Lambda^{N_c}$ for the $\langle u_r \rangle$ of (8), for these vacua there is a solution x^* to $P = \pm 2\Lambda^{N_c}$ and $P' = 0$. The vacua (8) thus parameterize the singularities of the curve $y^2 = \det(x - \Phi)^2 - 4\Lambda^{2N_c}$, re-deriving the results of [6,5]. As before, it is possible to show that this curve is obtained iff $W_\Delta = 0$.

These considerations can also be applied to $N = 2$ theories with hypermultiplets, where they can be written as superpotentials on the branch in which a single photon is confined. We first consider $N = 1$ $SU(N_c)$ gauge theory with adjoint Φ, a fundamental flavor Q and $\tilde{Q}$, and tree-level superpotential

$$W_{tree} = \lambda Q \Phi \tilde{Q} + m_Q Q \tilde{Q} + \sum_r \frac{g_r}{r} \mathrm{tr} \Phi^r; \tag{9}$$

for $\lambda = 1$ and $g_r = 0$, this would be $N = 2$, $SU(N_c)$ with a fundamental hypermultiplet of mass m_Q. For $\lambda \neq 0$, the first term leads to a potential which lifts those moduli associated with "mixed" gauge invariants, such as $Q\Phi^r \tilde{Q}$ or $\prod_{i=1}^{N_c} (\Phi^{r_i} Q)$, which involve both Φ and Q or $\tilde{Q}$. Because we want $\lambda \neq 0$ to have an Abelian Coulomb phase, such mixed gauge invariants are thus not moduli and there is no need to couple them to sources.

For $g_r \neq 0$, we get a variety of classical vacua; again, we focus on the vacua with unbroken $SU(2) \times U(1)^{N_c-2}$. The low energy theory in such vacua is $N = 1$, $SU(2)$ with one flavor and $N_c - 2$ decoupled photons. As in (6), the scale Λ_2 of the low energy $SU(2)$ theory is related to the scale Λ of the original high energy theory by $\Lambda_2^5 = g_{N_c}^2 \Lambda^{2N_c-1}$. The superpotential of this low energy theory is

$$W_L(\lambda, g, X) = W_{cl}(\lambda, g, X) + \frac{g_{N_c}^2 \Lambda^{2N_c-1}}{X}, \tag{10}$$

where $W_{cl}(\lambda, g, X)$ is the classical superpotential (9) evaluated in this vacuum, with $X = Q\tilde{Q}$, and the second term is the dynamically generated superpotential for $N = 1$, $SU(2)$ with one flavor. Again, we are assuming that $W_\Delta = 0$. By performing a Legendre transform from λ to the conjugate variable $Z = Q\Phi\tilde{Q}$ and from the g_r to the conjugate variables u_r, we obtain the superpotential

$$W = -\frac{X}{4\Lambda^{2N_c-1}} P^2(x = ZX^{-1}; u_r) + \lambda Z + m_Q X + \sum_r g_r u_r, \tag{11}$$

where $P(x; u_r) = \det(x - \Phi)$.

The quantum vacua with one photon confined and $N_c - 2$ left unconfined are given by the vacua of the superpotential (11). These vacua have

$$P^2 = 4\Lambda^{2N_c-1}(m_Q + \lambda x), \qquad P\frac{\partial P}{\partial x} = 2\lambda\Lambda^{2N_c-1}, \tag{12}$$

along with some additional equations, which combine to give two vacua for each set of the parameters. We see from (12) that these vacua lie on the singularity manifold of the hyper-elliptic curve

$$y^2 = P^2 - 4\Lambda^{2N_c-1}(m_Q + \lambda x). \tag{13}$$

By varying the parameters g_r, the vacua obtained above span the entire singularity manifold of the curve (13). The overall scale of the g_r is the order parameter for confinement. For any value of this overall scale, the projective space of the ratios spans the singularity manifold of the curve. Taking the overall scale to zero while holding the ratios fixed, we approach the transition points from the confining to the Coulomb phase[1]. Because these vacua are solutions of (2), the massless dyons of the original theory occur on the singularity submanifold of the curve (13). We thus *derive* the curve (13) for the photon coupling of the original theory via the superpotential (11) of the branch with a single confined photon and find agreement with the curves found in [10,11].

The generalization of (11) to the branch with a single confined photon for $N = 1$, $SU(N_c)$ with adjoint Φ and $N_f < N_c$ flavors is similarly found to be

$$W = -N_f \left(\frac{\det_{N_f} X}{4\Lambda^{2N_c-N_f}} [\mathrm{tr}_{N_f} P_{N_c}(x; u_r)]^2 \right)^{1/N_f} + \sum_{r=2}^{N_c} g_r u_r + \mathrm{tr}_{N_f} mX + \mathrm{tr}_{N_f} \lambda Z, \tag{14}$$

[1] Although the original theory is $N = 2$ supersymmetric only for $\lambda = 1$, there is a Coulomb phase described by the curve (13) for arbitrary λ.

with the $N_f - 1$ constraints:

$$\mathrm{tr}_n P_{N_c - N_f + n}(x; u_r - \mathrm{tr}_{N_f - n} x^r / r) = 0, \qquad n = 1, ... N_f - 1. \tag{15}$$

In these expressions, $X_i^{\tilde{j}} = Q_i \tilde{Q}^{\tilde{j}}$, $Z_i^{\tilde{j}} = Q_i \Phi \tilde{Q}^{\tilde{j}}$, $x \equiv Z X^{-1}$ is an $N_f \times N_f$ matrix, and $P_m(x; v_r = \mathrm{tr} M^r / r) \equiv \det_m (x 1_{m \times m} - M)$ and $\mathrm{tr}_{N_f} = \mathrm{tr}_n + \mathrm{tr}_{N_f - n}$. This result can be used to study the vacuum structure of the theory.

The technique of using an effective superpotential in the phase with a confined photon can be applied to determine the monopole structure of any theory with a Coulomb phase. However, there can be subtleties which make it more difficult than the above examples to obtain the exact effective superpotential. For example, as discussed in [22], when the index of the embedding of the unbroken $SU(2)$ in the original gauge group is larger than one, instantons in the broken part of the gauge group can contribute to the effective superpotential. Also, it is possible that $W_\Delta \neq 0$ in some cases. A better understanding of such terms is needed in order to use the confining phase superpotential technique to obtain the monopole structure of other theories.

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